Indian Statistical Institute
First Semester Back Paper Exam 2006-2007
M.Math.II Year
Graph Theory and Combinatorics

Time: 3 hrs | Date: -01-07 | Max. Marks : 100

Instructor: N S N Sastry

Answer questions upto a maximum of 100 marks.

1. Define a strongly regular graph. Compute the parameters of the strongly regular graph whose set of verticies is the finite field $\mathbb{F}_q, q \equiv 1(\text{mod}4)$ and two verticies are adjacent if their difference is a square. [12]

2. Define a bipartite graph. Show that the minimum eigenvalue of a finite, connected, k-regular graph is $-k$ if and only if the graph has no circuit of odd length. [10]

3. (a) Define the Cayley graph $\mathcal{G}(G, S)$ of a pair (G, S), where G is a group and S is a subset of G.

   (b) If G is free group on a set S, then show that the associated Cayley graph $\mathcal{G}(G, S \cup S^{-1})$ is a regular tree.

   (c) Assume that S generates G. Show that $\mathcal{G}(G, S)$ is a bipartite graph if and only if there exists a group homomorphism $\varphi : G \to \{\pm 1\}$ such that $\varphi(S) = -1$. $[2 + 6 + 6]$

4. Define the chromatic number $\chi(X)$ of a k-regular, connected graph X on n verticies. Show that $\chi(X)$ is at least $k(\max\{|\mu_1|, |\mu_{n-1}|\})^{-1}$ where $\{k \geq \mu_1 \geq \ldots \geq \mu_{n-1}\}$ is the spectrum of X. $[3 + 10]$

5. Determine the eigenvalues and their multiplicities of a finite strongly regular graph. Show that a strongly regular graph with parameters $n = 6u - 3, k = 2u, \lambda = 1, \mu = u(u \geq 1$, an integer) exist only when $u + 1$ divides 12. $[10 + 5]$

6. If L_1 and L_2 are two 3-subsets of $P = \{1, 2, 3, 4, 5, 6, 7\}$ such that (P, L_1) and (P, L_2) are projective planes of order 2 and $|L_1 \cap L_2| \geq 2$, show that $g(L_1) = L_2$ for some g in the alternating group A_7. [10]

7. Show that a projective plane of order 3 is unique. [12]

8. Define the weight enumerator of a q-ary code. Establish a relation between the weight enumerator polynomials of a q-ary linear code and of its dual.

[4 + 10]

9. Define an extendable 2-(v, k, λ) design. Show that a projective plane of order 7 cannot be extended. [3 + 7]

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